Article

On Hidden Mathematics in the Artwork of the Indigenous People of Brazil

Anja Noëmi Stubbe and Hannes Stubbe *

Humanwissenschafliche Fakultät (Faculty of Human Sciences), Herbert-Lewin-Str. 2, 50931 Cologne, Germany; anjanoemi@hotmail.com (A.N.S.)

* Corresponding author. E-mail: hstubbe@uni-koeln.de (H.S.)

Received: 11 December 2024; Accepted: 17 March 2025; Available online: 2 April 2025

ABSTRACT: Using examples from indigenous art in Brazil, this paper demonstrates that these works contain "hidden mathematics" (such as symmetry, striped ornaments, rows, *etc.*), which challenges the traditional notion of "primitive art".

Keywords: Art; Ethnomathematics; Indigenous art; Geometric patterns; Mathematical aesthetics

1. Introduction

Empirical, partly quantifying and experimental research in aesthetics only began with the studies of the natural philosopher and psychophysicist *Gustav Theodor Fechner* (1801–1887) in the 19th century. Fechner published various aesthetic problems from 1839 onwards and dealt, among other things, with the golden section and questions of authenticity (e.g., Holbein's Madonna 1865, 1866ff) (cf. [1] Altmann, 1995; [2] Arruda Leal Ferreira, 2021).

The implicit mathematics hidden in (living) nature and culture has since been deciphered in various ways. For example, snowflakes ([3] cf. Stewart, 2017), "crystal souls" ([4] Haeckel, 1917), phyllotaxy ([5] Coxeter, 1963:209-213), symmetry ([6] cf. Weyl, 1955), Gothic measurements ([7] Weber, 1954) and the golden section ([8] cf. Pfeifer, 1885; [5] Coxeter, 1963:199–209; [9] Beutelspacher & Petri, 1996; [10] van der Schoot, 2016) have received increased attention. The photographer Karl Bloßfeldt (1865–1932) photographed a variety of natural art forms, e.g., plant morphology, in his pictorial works "Urformen der Kunst" and "Wundergarten der Natur" ([11] cf. Bloßfeldt, 1981). *Karl Menninger* (1898–1963) commented on this question in his small paper "Mathematik und Kunst" (1959) as follows:

> "Mathematics and art are related in nature. Both revolve around the mystery of form. In it they touch each other: no art without form. All form, however, is based in the deepest sense on mathematical form". ([12] Menninger, 1959: 6)

In her introductory work "Maß und Zahl im Kunstwerk", *Ellen Weber* (1954) states:

> "Forms arise when chaotic things are brought into order. Order, however, is always moderate. A rule can belong to this measure, in which case the forms are regular. There is also the irregular form, which contains measures but no rules. Works of art are often composed of irregular forms. The measure is determined by the number. Thus, there is a number in every work of art, visible always in symmetry, often in harmony, but also invisible or at least difficult to recognise." ([7] Weber, 1954:1)

The Brunswick architect Fr. Wilhelm Kraemer ([13] 1965:4) shows that only a small canon of basic figures is available to humans in this world:

> "*linear*: perpendicular, horizontal, oblique and curve
> *two-dimensional*: triangle, rectangle, trapezoid, polygon, circle, ellipse, parabola and combinations of these shapes
> *cubic*: cuboid, prism, pyramid, cylinder, cone, sphere and their combinations".

All these basic forms occur in nature and culture, which *Kraemer* demonstrates plausibly with a large number of illustrations.

In his richly illustrated writing "The Beauty of the Snowflake", *Ian Stewart* (2017) states:

"Mathematics is a systematic, partially conscious technique that we invented to put our highly developed mental eye for patterns to good use. So it only makes sense to expect a strong connection between mathematics and beauty." ([3] Stewart, 2017:101)

In anthropology, [14] *Karl von den Steinen* (1925), who was significant for anthropology in Brazil, dealt with the expressive tattooing of the Marquesans at an early stage and offered interpretations. Also "the Dutch physician [15] *A.W. Nieuwenhuis* (1913/23), who lived for about five years with the Dajaks on Borneo, presented various ethnopsychological studies on the mental disposition (imagination, memory, logical thinking) of the 'Malay peoples of the East Indian archipelago' (=Dajaks and Torajas) in the years 1913 to 1923 in which he interpreted their material products ("Gebilde", creations) as the results of cognitive 'achievements'. The cultural achievements from which Nieuwenhuis deduced the mental disposition of these Malay ethnic groups were handicrafts, technology, and belief in spirits. The researcher concluded that the Malays were indeed 'culturally capable', not according to their actual level of development, but according to their developmental possibilities, and that "the mental disposition of the Malays is not inferior to that of the white race in the characteristics which have been investigated up to now" ([16] Stubbe, 2012:510). This can also be said of the Indians of Brazil. Among the indigenous people of Brazil, there is no actual word for "art" ([17] cf. B. Ribeiro, 1989:15f). What is really interesting in these cultures is the artistic action itself, not so much the product ([18] cf. Carpenter, 1971:165 in: Otten, 1971).

2. On Implicit Mathematics in Indigenous Art in Brazil

We select some mathematical aspects from indigenous art, such as symmetry, golden ratio, harmony and sequence, neglecting the close connections of indigenous art to myths and rituals, as well as to the natural environment in the most biodiverse country on earth ([19] cf. Natural History Museum Vienna, 2022:3).

2.1. Symmetry

Symmetry (from Greek σύμμετρος congruent) in mathematics means:

"Symmetry refers to certain regular properties of figures based on congruence. In the plane, symmetry is usually understood as axial symmetry. In addition, there is point symmetry and rotational symmetry. In space, there is point symmetry, axial symmetry and plane symmetry. Symmetry is also an artistic point of view and at the same time an important building principle of nature." ([20] Lexikon der Schulmathematik, 1980:1032)

In his book "Die mathematische Denkweise" (The Mathematical Way of Thinking), *Andreas Speiser* writes "Über Symmetrie in der Ornamentik" (On Symmetries in Ornamentation):

"The expression 'symmetry' means harmonising different parts of a whole. Symmetries are found everywhere where the spiritual manifests itself in matter. They already appear in nature; many living beings possess the special symmetry of mirror images, and plants largely exhibit rotational symmetries, particularly visible in the positions of leaves, blossoms, *etc.* For art, they form the element of life, and one can point them out in poetry and prose, in music and in painting." ([21] Speiser, 1952:17)

Speiser also rightly emphasises the connection with the mathematical group theory:

"The real reason why the symmetries are mathematically important lies in the group quality: if two covering operations are carried out in succession, a third is produced, or, to put it another way, two symmetries occurring simultaneously on the ornament condition another symmetry." ([21] Speiser, 1952:22)

Hermann Weyl (1955) [6], who dedicated an excellent, richly illustrated monograph to symmetry, writes:

"If I am not mistaken, the word *symmetry* is used in two senses in our colloquial language. In one sense, symmetrical means something like well-proportioned, balanced, and symmetry denotes that kind of concordance of several parts by which they unite to form a whole. Beauty is closely connected with symmetry." ([6] Weyl, 1955:11)

Weyl distinguishes between different types of symmetry: bilateral, translational, rotational, ornamental and crystallographic symmetry.

For the astrophysicist [22] *Mario Livio* (2010:305), symmetries are "a measure of the invariability of a system with respect to certain changes of place, time and orientation".

Examples from indigenous art in Brazil may illustrate the importance of symmetry (Figures 1–4):

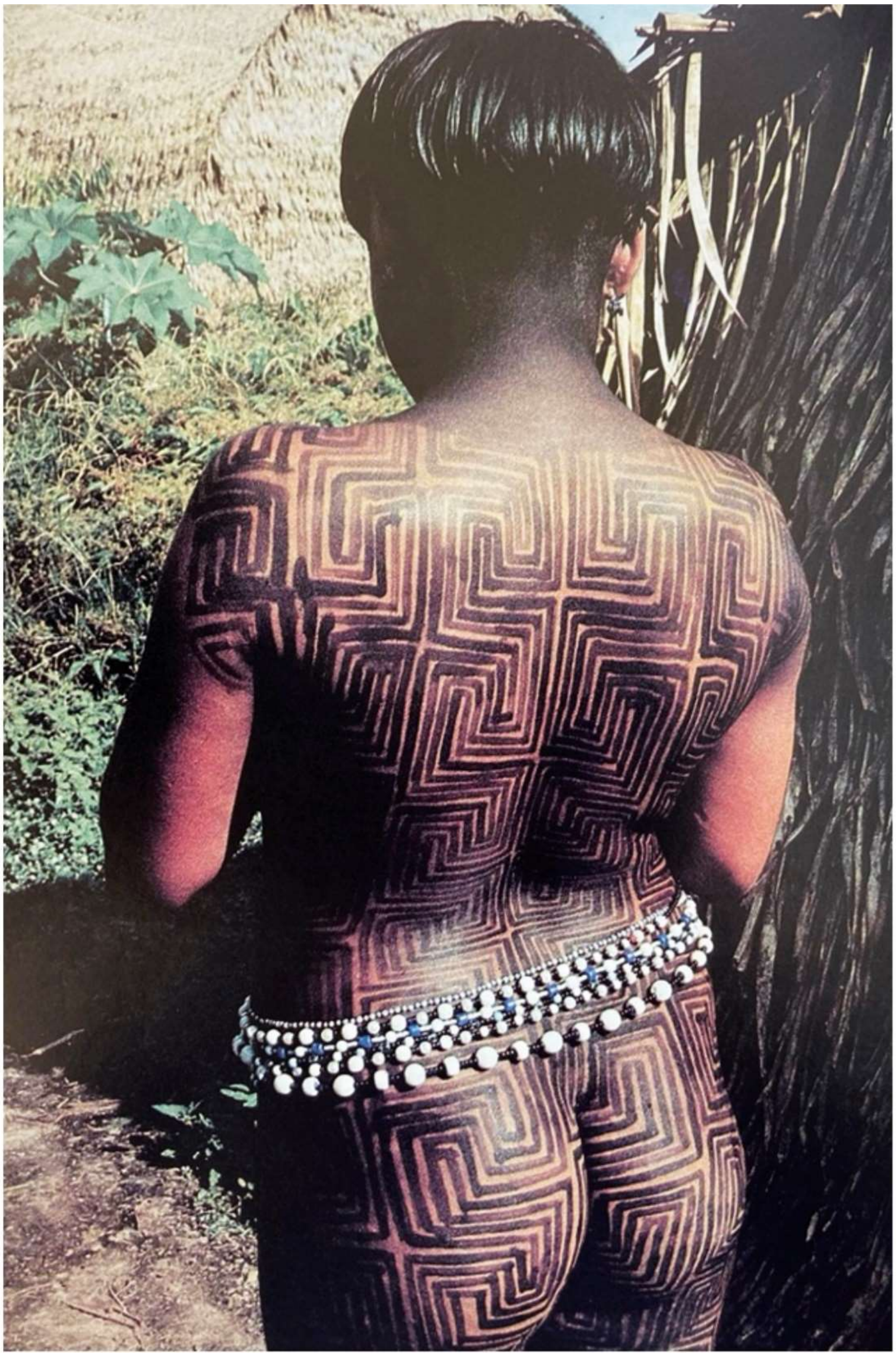


Figure 1. Body painting (Asurini do Xingú) (motivo *ipirajuak*), [23] Source: Vidal, 1992:235.

Figure 2. Fan (Abano indígena, Apalay, Pará), Source: ©Photo and property H. Stubbe, 2023.

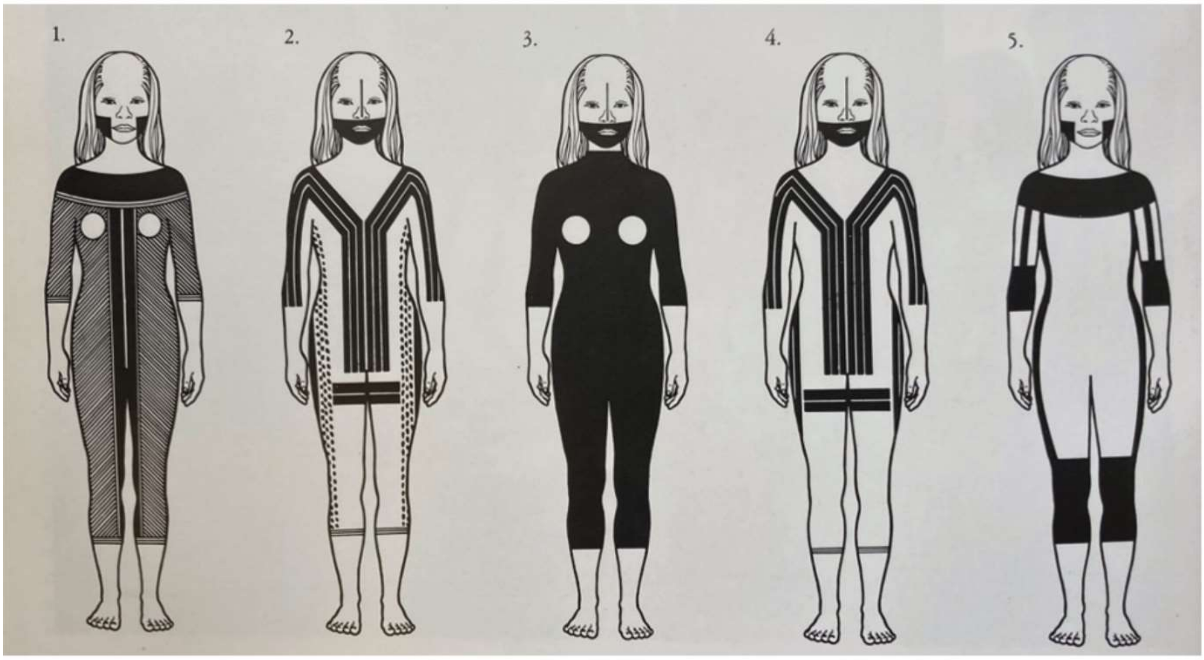


Figure 3. Female body painting (Cayapó-Xikrin do Cateté), (Pintura corporal feminina), [23] Source: Vidal, 1992:160.

A modern indigenous artist Xohã Karajá paints the following hanging picture with seven diamonds in a row:

Figure 4. *Untitled*. Colours on canvas. Private collection. [24] Source: ©Xohã Karajá.

2.2. Golden Section or Golden Ratio (and Fibonacci Numbers)

The term "golden section" or "golden ratio" is relatively new and only appeared in the 19th century. In European antiquity, there was apparently no concise term for it, and the Latin translators of Euclid used the paraphrase "proportio habens medium et duo extrema". *Luca Pacioli* (1497) then created the term "divina proportio" at the beginning of the 16th century, which goes back to a number-mystical attitude of the Middle Ages.

A mathematical school encyclopaedia defines:

> "Golden section (medial section; German Goldener Schnitt; French section d'or). For a distance divided in the golden section, the partial distances are
> a/2(√5 − 1) approximately 0.618a and
> a/2(3 − √5) approximately 0.382a "([20] Lexikon der Schulmathematik, 1980:350)"

The fan shown above (see Figure 2), for example, corresponds approximately to the golden ratio (27:43.5 = 0.62) in terms of its width/length proportion. In which indigenous objects can the golden ratio be found? I would be happy to leave the thorough examination of the corresponding ethnographic collections in the museums to my colleagues. The equation of the golden section with 'beauty' is disputed ([10] cf. Schoot, 2016), which is an interesting sentiment, but it is not fully developed in connection to the artworks.

The Fibonacci numbers, named after *Leonardo of Pisa* (gen. Fibonacci) (Liber abaci, 1202), are closely related to the golden section ([5] cf. Coxeter, 1963: 205ff; [22] Livio, 2010:285ff). An example of the Fibonacci number sequence:

1,1,2,3,5,8,13,21,34,55,89,144…

If you divide 89/144 = 0.618, for example, you get the golden ratio.

2.3. Stripe Ornaments and Rows

Stripe ornaments are ornaments that run between two boundary lines (Figure 5). There are four covering operations for them: 1. translations in the direction of the longitudinal axis (parallel to the boundary straight lines), 2. mirroring and sliding mirroring on the longitudinal axis, 3. mirroring on transverse axes, 4. rotations about a point on the longitudinal axis ([7] cf. Weber, 1954:40).

> "For all ornaments, it is true that there is a piece of the ornament by whose translation the whole ornament is produced, the so-called elementary ornament (ornamental atom). The period of translation is called the elementary distance of the ornament." ([7] Weber, 1954:39)

With regard to shapes, a distinction can be made between *geometric* and *vegetal ornamentation*. A geometric ornament is created when geometric forms are combined to comply with certain mathematical laws. Vegetable ornamentation contains the same mathematical forms of order, but its partial forms are taken from the plant and animal kingdoms, e.g., skin patterning ([24] cf. Zerries, 1965).

As forms of order in ornamentation, a distinction is made between rhythmic sequence, also called translation (a-a-a...) and compound rhythmic sequence (ab-ab-ab-ab...), axial symmetry (reflection on a straight line) and central symmetry (reflection at a point).

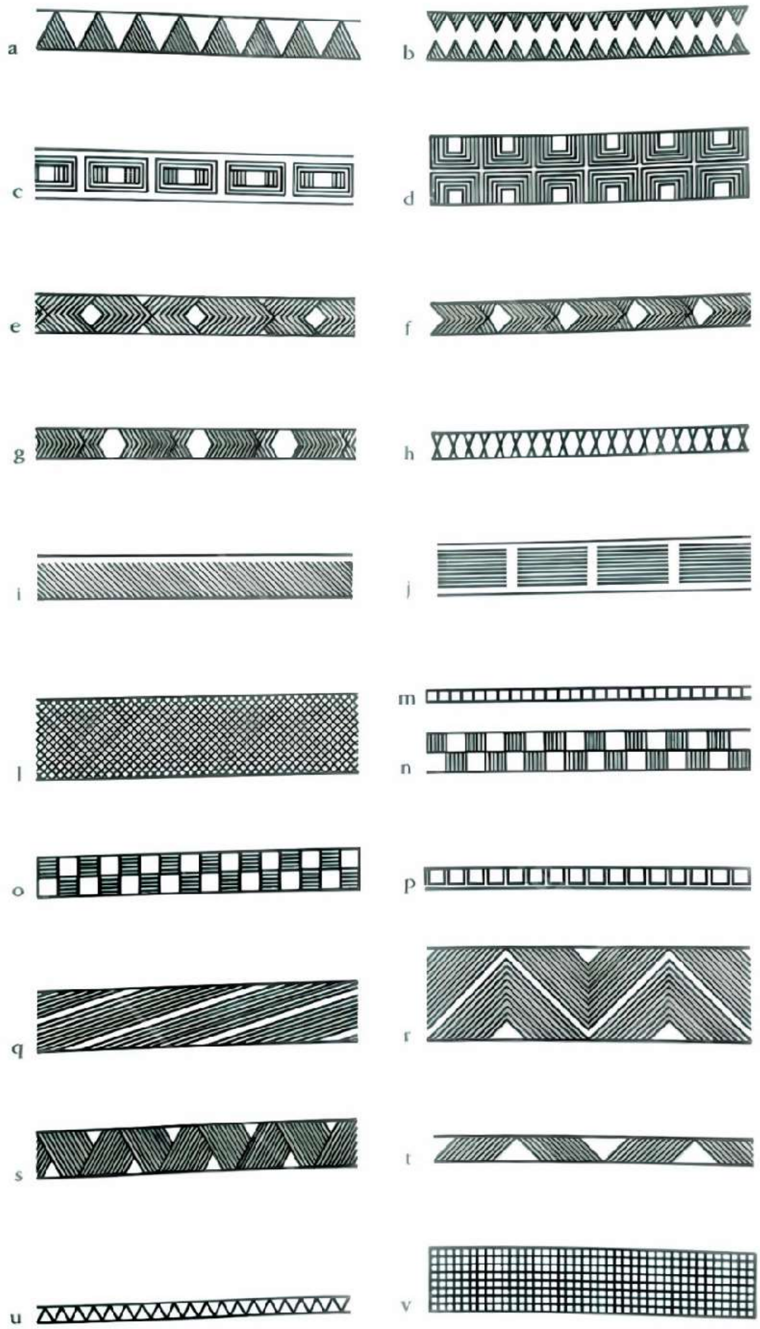


Figure 5. (**a–j**,**l–v**) Decorative striped ornaments of the face (Kayapó-Xikrin), [23] Source: Vidal, 1992:151.

A well-known ornament, already known in prehistoric times, is the *meander* (Figure 6), consisting of right-angled broken and intertwined lines ([7] cf. Weber, 1954:47), as they are also found in ancient Greek, Chinese and Mexican ornamentation.

Figure 6. Loom with loincloth made of small beads (Tiriyó, Pará), [25] Source: Martius-Staden Jahrbuch (São Paulo), 1999/2000:120.

It is an ornamental band, broken at right angles, which derives its name from the meandering river Maiandros in Asia Minor. It goes back to an ancient weaving and braiding pattern. A distinction is made between the hooked meander band, the concentric meander band and the spiral meander band.

The technique of braiding results in the *braided ornament* or the intertwined ornament.

In mathematics, a *"series"* is understood to be a sequence of finite or infinite links. A distinction is made between infinite series, where the links can be numbers or functions. In the case of convergent series, the sequence of their partial sums converges, *i.e.*, has a limit value, otherwise, one speaks of divergent series.

2.4. Harmony and True-to-Size Proportions

In the European aesthetic tradition, "harmony" is understood to mean order, evenness, and agreement of all parts of a phenomenon. It is "the harmony of dissimilar or opposing elements into a balanced overall form" ([26] Lexikon der Ästhetik, 2004:150). The sculptor *Polyklet* (5th century BC) attempted to standardise the beauty of the human body by elaborating an ideal figure, which he called "kánon" (κανων). In his theory of proportion, which was received in the Renaissance, the length of the head is 1/8 of the length of the body. Since the sphere was the perfect mathematical body for the Greek natural philosophers, they depicted the world as a "cosmos", *i.e.*, as a clearly constructed, beautiful (adorned), ordered, harmonious world. Does this concept of harmony also apply to indigenous artistic creation?

> "It seems to be proven that the concept of beauty starts from very regular, symmetrical forms and develops towards more and more complex entities. Children, for example, find axis-symmetrical drawings particularly successful, while adults find the same symmetry rather boring. In the historical development of art, too, one can observe the 'progression' from simple and manageable to complex and differentiated forms." ([9] Beutelspacher & Petri, 1996:171f)

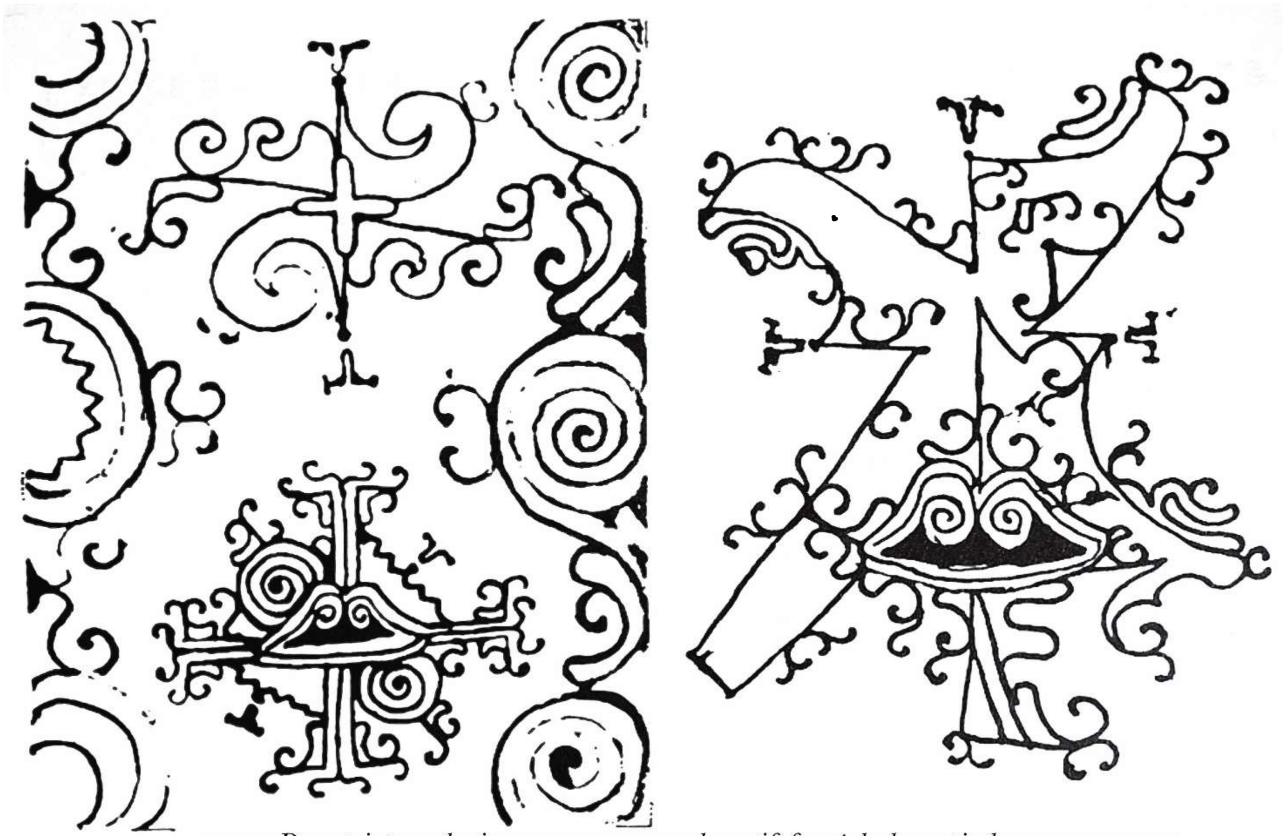
A large number of Brazilian indigenous art objects are also considered extraordinarily beautiful, harmonious and attractive to the European eye. What is the reason for this? Can this general aesthetic sensation be explained by the "psychic unity of humanity" ([27] cf. Stubbe, 2021:192-198)? Is the "book of nature" written in the language of mathematics ([22] Livio, 2010)?

3. Other Geometric Forms

Spiral forms ([6] cf. e.g., Weyl, 1955:74ff; [5] Coxeter, 1963:203ff) can already be found in archaic cultures, e.g., in Neolithic and megalithic Europe ([28] cf. Walker, 1997:36; [29] Stubbe & Frenken, 2023:57) or as geoglyphs in the Nasca desert ([30] cf. Schliephacke, 1979:58f; [31] Koch, 2019), as well as in indigenous art in Brazil (Figure 7).

> "A spiral is a curve that circles a central starting point over and over again, getting further and further away from it in one direction while getting closer and closer to it in the other. There are many types of spirals, but only one of them, called the logarithmic spiral, shows precise scale symmetry." ([3] Stewart, 2017:122)

In the Archimedean spiral, the successive turns have the same distances. In nature, the best-known examples of a logarithmic spiral are the nautilus shell and the fossil ammonites.



Deux peintures de visage; on remarquera le motif formé de deux spirales affrontées qui représente — et s'applique sur — la lèvre supérieure.

Figure 7. Face paintings (Caduveo) featuring a motif of two opposing spirals representing the upper lip. [32] Source: Cl. L. Strauss, 1955:213.

On the basis of the above, we can state that the "creations" ([33] "Gebilde", Bühler, 1927), *i.e.*, the products produced by humans, here the art objects of the Brazilian Indians, can also be subjected to mathematical analysis. What conclusions can be drawn from this about the cognition, perception and visual thinking of Brazilian indigenous people? Is this hidden mathematics or geometry in the artworks of the indigenous people innate and universal, or is it copied from nature? In any case, one can no longer speak of "primitive art" as before. The illustrated presentation has tried to make this very clear.

Further reading is presented in Appendix Table A1.

Appendix A

Table A1. Further Reading.

Number	Reference
1	Baldus H. Os carimbos dos índios do Brasil. *Revista do Museu Paulista (São Paulo),* **1962**, *13*, 7–74.
2	Berliner A. Geometrisch ästhetische Untersuchungen mit Japanern und an japanischem Material. *Arch. für die gesamte Psychologie,* 1924.
3	Boelitz Vidal L. A pintura corporal entre índios brasileiros. *Revista de Antropologia,* **1978**, *21*, 87–93.
4	Costa MHF. *A arte e o artista na sociedade Karajá*; FUNAI: Brasília, Brazil, 1978.
5	Dorta SF, MX. *A plumária indígena brasileira*; edusp: São Paulo, Brazil, 2000.
6	Einstein A. *Mein Weltbild*; Ullstein: Berlin, Germany, 1988; S.119.
7	Fechner GT. *Vorschule der Aesthetik. 2 Thle*; Breitkopf & Härtel: Leipzig, Germany, 1876.
8	Fechner GT. Zur experimentellen Aesthetik. Thl. 1. In *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, Math.-phys. Cl. Bd. 9*; Hirzel: Leipzig, Germany; 1987; pp. 553–635
9	Gröning K. *Geschmückte Haut. Eine Kulturgeschichte der Körperkunst*; Frederking & Thaler: München, Germany, 1997.
10	Hartmann G. *Masken südamerikanischer Naturvölker*; Museum für Völkerkunde Berlin: Berlin, Germany, 1967.
11	Hartmann G. *Schmuck der Indianer Südamerikas*; Landesmuseum Koblenz: Koblenz, Germany, 1981.
12	Karsten R. *Body painting and tattooing in South America*; IPEK, Leipzig, Germany, 1925; pp. 151–164.
13	Kreide-Damani I. *Kunst-Ethnologie. Zum Verständnis fremder Kunst*; Dumont: Köln, Germany, 1992.
14	Literatura e arte indígena no Brasil. *Cadernos do Instituto Luso-Brasileiro 2*. Universität zu Köln: Köln, Germany, 2022.
15	Melatti JC. *Índios do Brasil*; Editora da USP: São Paulo, Germany, 2007.
16	Murdock GP. Cross sex patterns of skin behaviour. *Ethnology* **1971**, *10*, 359–368.
17	Musée d'Ethnographie (Genève). *L'art de la plume brésil*; Musée d'Ethnographie: Genève, Switzerland, 1985.
18	Ribeiro B. *Dicionário do artesanato indígena*; Ed. da Universidade de São Paulo: São Paulo, Germany, 1988.
19	Ribeiro D. *Suma etnológica brasileira. Arte índia*; Vozes: Petrópolis, Brazil, 1986; Volum 3.
20	von den Steinen K. *Unter den Naturvölkern Zentral-Brasiliens*; D. Reimer: Berlin, Germany, 1897.
21	Stubbe AN. *Kleine Kunstgeschichte Brasiliens. Eine wissenschaftliche Einführung*; Shaker: Aachen, Germany, 2018.
22	Stubbe H. Kulturanthropologische Aspekte der Trauer südamerikanischer Indianer. *Sociologus* **1990**, *40*, 54–68.
23	Stubbe H. *Curt Nimuendajús Bibliothek im tropischen Brasilien (1903–1945). Ein Beitrag zur Geschichte der Ethnologie*; Shaker: Düren, Germany, 2020.
24	Stubbe H. *Paläopsychologie. Eine kommentierte Bibliografie*; Shaker: Aachen, Germany, 2024.

Acknowledgments

The authors would like to thank everyone who contributed to the research and preparation of this article.

Author Contributions

Both authors contributed equally to this work.

Ethics Statement

Not applicable.

Informed Consent Statement

Not applicable.

Data Availability Statement

Data availability is not applicable.

Funding

This research received no external funding.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. Altmann I. *Bibliographie Gustav Theodor Fechner*; Verlag im Wissenschaftszentrum: Leipzig, Germany, 1995.
2. Elias Morelli Ribeiro A, da Rosa HLRS, do Amaral Gama Santos Marcus V. (Organisadores). *Para além da psicofísica. Fechner e as visões diurna e noturna*; NAU: Rio de Janeiro, Brazil, 2021.
3. Stewart I. *Die Schönheit der Schneeflocke*; WBG: Darmstadt: Germany, 2017.
4. Haeckel E. *Kristallseelen. Studien über das anorganische Leben*; Kröner: Leipzig, Germany, 1917; 3. Aufl.
5. Coxeter HSM. *Unvergängliche Geometrie*; Birkhäuser: Basel, Switzerland, 1963.
6. Weyl H. *Symmetrie*; Birkhäuser: Basel, Switzerland, 1955.
7. Weber E. *Mass und Zahl im Kunstwerk*; Vieweg: Braunschweig, Germany, 1954.
8. Pfeifer FX. *Der Goldene Schnitt und dessen Erscheinungsformen in Mathematik, Natur und Kunst*; Sändig: Vaduz, Liechtenstein, 1885. (Reprint 2014).
9. Beutelspacher A, Petri B. *Der Goldene Schnitt*, 2nd ed.; Spektrum: Heidelberg, Germany, 1996.
10. van der Schoot A. *Die Geschichte des goldenen Schnitts. Aufstieg und Fall der göttlichen Proportion*; frommann-holzboog: Stuttgart, Germany, 2016.
11. Bloßfeldt K. *Das fotographische Werk*; Schirmer & Mosel: München, Germany, 1981.
12. Menninger K. *Mathematik und Kunst*; V&R: Göttingen, Germany, 1959.
13. Kraemer FW. *Uns alle trägt die gleiche Schöpfung*; Materia Medica Nordmark: Hamburg, Germany, 1963.
14. von den Steinen K. *Die Marquesaner und ihre Kunst. Primitive Südseeornamentik. I. Tatauierung*; D. Reimer/E. Vohsen: Berlin, Germany, 1925. (Reprint 2008).
15. Nieuwenhuis AW. (1913–1923): Die Veranlagung der malaiischen Völker des ostindischen Archipels. *Internationales Archiv für Ethnographie*, 21,22,23,25.
16. Stubbe H. *Lexikon der Psychologischen Anthropologie*; Psychosozial: Gießen, Germany, 2012.
17. Ribeiro B. *Arte Indígena, linguagem visual*; Ed. da Universidade de São Paulo: São Paulo, Germany, 1989.
18. Otten CM. *Anthropology and Art. Readings in Cross-Cultural Aesthetics*; American Museum of Natural History: New York, NY, USA, 1971.
19. Naturhistorisches Museum Wien (Hrsg.). *Brasilien. 200 Jahre Beziehungsgeschichte*; nhm: Wien, Austria, 2022.
20. Athen H, Bruhn J. *Lexikon der Schulmathematik. 4 Bde*; Aulis, Deubner & Co.: Köln, Germany, 1980.
21. Speiser A. *Die mathematische Denkweise*; Birkhäuser: Basel, Switzerland, 1952.
22. Livio M. *Ist Gott ein Mathematiker? Warum das Buch der Natur in der Sprache der Mathematik geschrieben ist*; Beck: München, Germany, 2010.
23. Vidal L. (Org.). *Grafismo indígena*; EDUSP: São Paulo, Germany, 1992.
24. Zerries O. *Das Tier und seine Hautmusterung als Vorbild der Körperbemalung*; Bulletin Société Suisse des Américanistes: Genève, Switzerland, 1965; pp. 45–69.
25. Stubbe H. *Probleme der Ethnoästhetik brasilianischer Indianer*; Institut Martius-Staden. Jahrbuch: São Paulo, Germany, 1999/2000; Nr.47/48, pp. 121–130.
26. Henckmann W, Lotter K. *Lexikon der Ästhetik*; Beck: München, Germany, 2004.
27. Stubbe H. *Weltgeschichte der Psychologie. Eine Einführung*; Pabst: Lengerich, Germany, 2021.
28. Walker B. *Die geheimen Symbole der Frauen*; Hugendubel: München, Germany, 1997.
29. Stubbe H, Frenken R. *Paläopsychologie*; Pabst: Lengerich, Germany, 2023.
30. Schliephacke BP. *Bildersprache der Seele. Lexikon der Symbolpsychologie*; Telos: Berlin, Germany, 1979.
31. Koch M. Eine Wüste voller Bilder und Rätsel—die Nasca-Kultur in Peru. *Amerindian Res.* **2019**, *14*, 14–28.
32. Lévi-Strauss C. *Tristes tropiques*; Plon: Paris, France, 1955.
33. Bühler K. *Die Krise der Psychologie*; Gustav Fischer: Jena, Germany, 1927.